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HOMER HOYT'S URBAN EMPLOYMENT MULTIPLIER

by

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I. INTRODUCTION

This thesis is an investigation of an economic parameter for urban economies known as an export employment multiplier. It is referred to in this thesis as the Hoyt employment multiplier after the name of its principal innovator, Homer Hoyt. Two other employment multipliers are the investment employment multiplier of R. F. Kahn and the input-output employment multiplier of W. W. Leontief. The Leontief employment multiplier has been rather well developed, but the Kahn and particularly the Hoyt urban employment multipliers have not.

Although the idea of an urban export employment multiplier had already appeared, Homer Hoyt refined the concept in the late 1930's when he was the principal economist for the Federal Housing Administration. From their book of 1939, Weimar and Hoyt (76, p. 27) defined basic (export) activities to be those which commanded a stream of income from outside the city. Hoyt's method of predicting the future size of cities was to predict its future employment as a multiple of predicted basic employment. To identify basic (export) employment in any city he used the following method. All employment in manufacturing, mining, resorts and residences of retired people and nonlocal government was regarded as export. By type of activity, parts of transportation service employment could be designated export. Employment in schools with students whose parents lived outside of the city was designated as export. All other export employment was considered to occur in trade and finance.

Hoyt thought the problems of determining export employment in trade and finance by any direct method to be great, and used a residual method for them. From studies of the F.H.A. he concluded that one export employee gave rise to one nonexport employee. Taking all employment in a city as 100 per cent, he determined the percentage of export employment in all sectors but trade and finance and doubled it. The residual, if any, was the doubled per cent of export employment in trade and finance. Then, by estimating the future change in each of the export employments, he predicted the change in total employment. From anticipated total employment change he expected population to change accordingly and this gave him an indication as to whether real estate values would receive an upward or downward pressure from the local economy. The anticipated change in real estate value was his main concern as an F.H.A. economist.

Hoyt subsequently modified his export employment estimator, recognizing that not all of manufacturing need be for export, for example, and dropped the 1:1 ratio of export to nonexport employment. His additional efforts in this area were mainly in the sophistication of his export employment extimator. Although considerable work has been done by others in the development of urban income accounts and the input-output employment multiplier, to this writer's definition and knowledge, no rigorous comprehensive treatment of the urban export employment multiplier has been made.

To develop a rigorous comprehensive treatment of the export employment multiplier as it pertains to urban economies, collation has been made with the investment and input-output employment multiplier formulations as all three should be consistent. Since the proof of the cooking is in the pudding, some empirical data relating to an input-output study of the St. Louis metropolitan area for 1955 was secured from Professor Werner Hirsch (31) of Washington University. Hoyt and Leontief estimates were prepared from this data to confirm that the theoretical exposition of the Hoyt multiplier performed as expected.

The empirical implementation of an export employment multiplier model introduced consideration of estimators of the model and of export and total employment. The organization of the thesis follows this chronology, the next chapter developing the Kahn, Hoyt and Leontief employment multiplier models. In chapter III, the three employment multiplier models are collated in order to see what has to be specified in order that they be consistent. Chapter IV investigates sufficient conditions for two estimators, commonly used to estimate the Hoyt employment multiplier, to be in agreement with the export employment multiplier model. Chapter V investigates sufficient conditions for the survey estimator of export and total employment, used to implement the estimators of the Hoyt model discussed in Chapter IV, to be correct. Chapter VI presents Leontief and Hoyt employment multiplier estimates for the St. Louis metropolitan area. The next chapter concludes the thesis.

II. EMPLOYMENT MULTIPLIER MODELS

In order to appraise the urban employment multipliers of Kahn, Hoyt and Leontief it is necessary to set up a system of urban income accounts, an investment multiplier and an open input-output scheme, the latter two with well defined relationships to the income accounts. This is in order to know what is being talked about. With this preparatory work out of the way the employment multiplier models can be considered.

The first problem to be considered in compiling urban income accounts is differences in region of employment of productive factors and the region of residence of the owners of these factors. This problem can be resolved by distinguishing <u>income produced</u> in an area and <u>income received</u> in an area. This distinction has been formalized at the national level in United Nations studies as indicated by Easterlin (25, p. 26).

The difference between income produced and income received can be illustrated by considering value added. Following the earnings-flow approach inherent in value added, it measures payments to the four ultimate factors of production, including direct taxes, in the form of wages and salaries, rent, interest and profit, before depreciation and dividends, and represents income produced. This magnitude corresponds to national income produced plus depreciation. Income received has to be the identical amount for a world economy. Total income produced in the world is the result of factors employed in, or out of, any given urban area, whose owners reside either in, or out of, the area. World income received is taken by owners residing either in, or out of, any given urban area, who own factors that are employed either in, or out of, the urban area. These statements as to the geographical distribution of factors producing income and factor owners receiving income, with respect to any given area, will be put in the following form:

> income produced
>> by factors employed in-out of the area
>> whose owners reside in-out of the area
>
> income received
>> by owners residing in-out of the area
>> owning factors employed in-out of the area

The income produced or received combinations of interest for a given area are:

	Produced		Received
(1)	in in	(4)	in in
(2)	in out	(5)	in out
(3)	out in	(6)	out in

The (2) combination, for example, is read:

> income produced
>> by factors employed in the area
>> whose owners reside out of the area

The most meaningful urban area totals are the following income produced and received combinations:

(1) + (2) (4) + (6)	each equal to income produced
(1) + (3) (4) + (5)	each equal to income received
(3) - (2) (5) - (6)	each equal to net in-area factor owner flow

The difference between income received and income produced calculated from the produced combinations (1) + (3) - [(1) +

(2)] = (3) - (2) = net in-area factor owner flow. The same conclusion follows working with the received combinations. Therefore, urban income received = urban income produced + net in-area factor owner flow.

To amplify the expression, "net in-area factor owner flow", it consists of wages and salaries, rent, interest and gross profits received by area residents for their factor services out of the area minus payments to area factors whose owners reside out of the area. As discussed later, corporate profits, before dividends, can be apportioned so as to preclude the necessity for any area adjustment.

United States national income accounts are compiled on an income received by U.S. residents basis, as indicated by Samuelson (61, p. 233). That the specific income account, national income, does measure income received by residents of the U.S. is evidenced by the addition of the net in-area factor owner flow to net foreign investment in the income accounts as measured on a goods-flow basis. A familiar example is interest received by American investors from investments in foreign lands minus interest paid to foreign owners of American investments being added to the net foreign investment component of GNP. A similar procedure applies to other factor payments as well.

Considering national income for an urban area, denoted UI, it is composed of four elements corresponding to payments to the four ultimate factors of production.

Factor	Payment
labor	wages and salaries
land (natural resources)	rent
capital	interest
risk taker	profit

The problem raised by a difference in the area of employment of a factor and residence of its owner has been clarified. For purposes of parallel with national compilations, area income received could be worked with. Employed residents goes with income received while employment in an area goes with income produced. Since the empirical data to be worked with in this thesis are of the employed in the area-income produced-type, urban income created will be worked with. If indirect taxes and depreciation are added to factor payments paid, an approximation to gross product produced for an urban area is obtained. Gross product can, of course, be measured directly.

A second problem in the measurement of UI produced is the presence of supraregional transactors such as corporations. The main conceptual difficulty that they raise is the geographic distribution of corporate profits. The hypothetical solution offered here calls for the same allocation, before dividends, whether income produced or income received is being estimated. Corporation headoffice expenses are to be allocated to operating plants or facilities as a service performed and corporation profits before taxes and dividends are then allocated on a value-added basis. If the headoffice is out of the area it is an import of services for the plant in the area. If only the headoffice is in the area, it is a service exporter. If the headoffice and an operating plant are in the area, the plant and its share of the headoffice expense can be regarded as a vertically integrated firm. The problem of supraregional governmental units will be dealt with subsequently.

A. Income Accounts

Considering GNP produced for an urban area, which will be denoted GUP, it can be directly estimated by the value of the following types of goods-flow measured at market prices including indirect taxes.

<u>Goods and services</u>

consumption
local government purchases of input
gross investment, domestic
net foreign investment

The idea is to break down spending on these four types of goods and services so that their total value will correspond to UI produced plus indirect taxes plus depreciation.

Consumption good and service expenditures will be defined to consist of all such purchases by residents of the area considered. Mail order imports as well as tourist and nontourist consumption purchases out of the area by residents are then a negative component of net foreign investment as well as imports of intermediate goods and services. Imports of goods and services by the final area demander will be referred to as direct imports, all other imports will be referred to as intermediate. Intermediate imports consist of all imports into an urban area for local resale before the goods and services involved reach their area final demander. The reason for distinguishing intermediate imports is that only direct imports could be allocated to domestic investment, exports, consumption or government by survey without the preparation of an input-output table.

A criteria for residents and nonresidents is needed. Nationally, six months stay is enough to qualify a person as a resident of a country according to Kindleberger (41, p. 17).

In the national accounts all governmental units are grouped together. A distinctive feature of subnational income accounting is that government contribution to GUP is best handled by breaking it into three components: (1) supraregional government expenditures on goods and services, (2) regional (urban) and subregional government expenditures on goods and services and (3) purchases by government units without tax authority in the area (designated <u>other</u> regional governments). Supraregional government purchases minus their direct imports of goods and business services are put into net foreign investment along with exports to <u>other</u> regional governments. Regional and subregional government purchases of goods and business services, minus their direct imports, receive the appelation of local government contribution to GUP.

Gross investment, domestic, will be defined to consist of gross private capital formation which includes most expenditures on buildings (residential and commercial) and on commercial durable goods for equipment purposes, plus the net change in business inventories. This component is identified as regionally produced by the physical location of the buildings, equipment and inventories. Direct imports of goods and business services are deducted from the overall figures. One could classify domestic investment by residence of the owner of the funds used to finance the investment, money from nonresidents being included in net foreign investment. This is not done in the national income accounts since net foreign investment is limited to exports minus imports of goods and services plus net national factor owner payment in-flow and, as indicated by Boulding (17, pp. 282-283), plus net money gift in-flow. In this paper, domestic investment is as previously defined regardless of the investor's residence.

Exports, as here defined, include all goods and business service exports whether for consumption, investment, or further processing, to the private sector and sales to supraregional and <u>other</u> regional governmental units. Tourist and other nonresident consumption expenditures in the area are also included in exports. The export total is equal to the sum of these items minus their direct imports. Net foreign investment is this total minus direct consumption imports and all intermediate imports. All resident consumption purchases out of the area as tourists and nontourists, as well as their mail order imports, must be subtracted since they were included in consumption expenditures. Direct imports involved in domestic investment and local government expenditures were netted out of those figures.

B. Investment Multiplier

The objective now is to set up an investment multiplier model which will relate to GUP produced or some recognizable variation of it. The definitional identity of saving and investment in or out of equilibrium will be used as the starting point for setting up the investment multiplicand. Simplifying a bit, disposable income for an area will be indicated by a national income accounts type of tabular arrangement:

Income item	Symbol
Gross urban product produced	GUP
- depreciation	dep
Net urban product	NUP
- indirect business taxes	Tx
Urban income	UI
- corporate profits tax	Tx
- net corporate savings	NCS
- business social security taxes	Tx
+ local govt transfer payments	Tf
+ supragovt transfer payments	Tf
+ private in-area transfers to residents	in-Tf
- out-area transfers by local businesses	out-Tf
Personal income	PI
- personal taxes	Tx

Disposable income	DI
= consumption	C
+ net personal savings	NPS
+ out-area transfers by residents	out-Tf

All of the preceding items are self-explanatory except possibly the transfer items. Government transfer payments are those payments which are not for factor services, such as social security and relief. Federal government payments of interest on the national debt are, by national income accounting definition, regarded as transfer items. Private in-area transfer payments include in-area factor owner flow of payments to residents and receipt of gifts of money. Out-area transfers include all out-area factor owner flow and money gifts by residents to nonresidents.

Symbolically, disposable income can be written as

GUP
- dep
- all Tx
- NCS
- out-Tf (by businesses)
+ in-Tf
+ govt Tf

DI = C + NPS + out-Tf (by residents)

Or, GUP = dep + NCS + NPS + C + all Tx + out-Tf - in-Tf - govt Tf.

As defined in the "Income Accounts" section, gross urban product produced is equal to consumption plus investment plus local government purchases of input. Let local government contribution to gross urban product produced be denoted by "local G". Investment was broken down in the preceding section as follows:

Item	Symbol
Gross private capital formation	GPCF
+ inventory additions	inv add
- inventory depletions	inv dep
+ exports, including supragovt expenditures	E
- imports of consumption goods	M_c
- imports of intermediate goods	M_{inter}

Thus, GUP = C + GPCF + inv add - inv dep + E - M_c - M_{inter} + local G.

Equating the two expressions for GUP:

dep + NCS + NPS + C + all T_x + out-Tf - in-T_f - govt T_f + (local G - local G) = C + GPCF + inv add - inv dep + E - M_c - M_{inter} + local G.

Cancelling like terms leaves the definitional equality of savings and investment:

dep + NCS + NPS + all T_x + out-Tf - in-T_f - govt T_f - local G = GPCF + inv add - inv dep + E - M_c - M_{inter}.

Working towards the formulation of multiplier investment to be used in conjunction with the employment multipliers to be considered, investment can be thought of as GPCF + inv add + E + in-T_f + govt T_f + local G. From the previous definitional equality of savings and investment, savings must then consist of dep + NCS + NPS + all T_x + out-T_f + inv dep + M_c + M_{inter}. The inclusion of taxes, inventory depletions, M_c and M_{inter} in total savings is contrary to usual investment multiplier practice. The reason for this formulation is that it is in accord with input-output procedure in general and that of the St. Louis empirical input-output scheme in particular. Imports and gross inventory reductions could be treated as negative components of investment and this need not cause any difference in quantitative results. For advantages of exposition, however, imports and gross inventory reductions will be left as components of savings.

Equating a planned investment of magnitude N composed of the elements indicated in the preceding paragraph with a planned savings function, denoted by $s(Y)$, composed of the elements also indicated in the preceding paragraph gives $N = s(Y)$ which must be solved to find the equilibrium Y. Y will consist of C, local G, domestic investment (I_d) and net foreign investment (NFI) as defined in the urban income accounts plus all government transfers to residents plus private in-area transfers to residents. This can be shortened to Y = GUP produced + all government transfers + private in-area transfers to residents.

An investment multiplier is concerned with changes in Y caused by changes in investment (I) found by multiplying ΔI times the reciprocal of the marginal propensity to save (MPS). The validity of this procedure is shown in the following derivation. For equilibrium Y_o, $s(Y)$ = planned I and for a change in I such as $I_o + \Delta I = I'$ then $s(Y) = I'$ which is the same as saying $\Delta[s(Y)] = \Delta I$. Now $\Delta s(Y) \cong \frac{ds(Y)}{dY} \Delta Y$, denoted $(MPS)\Delta Y$. For equilibrium, $(MPS)\Delta Y$ is set equal to ΔI; solving for the unknown, $\Delta\hat{Y} = \Delta I(\frac{1}{MPS})$ where the latter factor is termed the "investment multiplier".

If $s(Y)$ is a linear polynomial in Y, such as $L + \alpha Y$, then the equilibrium Y is found by equating planned investment (N) and saving as before. The solution is quite simple in this case, $Y = (N - L)/\alpha$ where α is the universal MPS and $\Delta Y = \frac{\Delta(N - L)}{\alpha}$ is usually restricted to $\frac{\Delta N}{\alpha}$.

Making the transition to the investment account to be used in considering employment multipliers, a modification has to be made. It stems from the fact that employment in an area is not <u>directly</u> affected by two components of the previous investment account, GPCF + inventory additions + all govt T_f + local G + E + private in-area T_f to residents. There is no employment at all associated with government transfers and private in-area transfers to residents excluding in-area factor owner flow to residents. There will be employed residents for most or all of net area factor owner flow but not any persons employed <u>in</u> the area. These transfer items are part of disposable income and do cause respending which is area income produced and cause consumption employment. They will be omitted from the investment multiplier model even though they cause consumption employment, since they do not cause investment employment and hence an investment employment multiplier is undefined. Government transfers and private in-area transfers to residents will henceforth be referred to as <u>transfers</u>, the urban income and employment stemming from them via multiplier analysis will be referred to as transfer derivative income and transfer derivative employment. The previously defined investment used in conjunction with the investment multiplier minus the transfers will be referred to as multiplier investment.

Equating a savings function of Y with a multiplier investment account consisting of GPCF + inventory additions + local G + exports will yield GUP produced - T_f (investment multiplier - 1) = GUP produced - transfer derivative income, denoted Y'. Changes in GUP produced - transfer derivative income can be estimated by the product of changes in the corresponding investment account times the reciprocal of MPS, where the MPS is evaluated at the corresponding GUP produced + transfer (Y) amount.

C. Kahn Multiplier

Attention can now be directed to an evaluation of the employment multipliers of Kahn and Hoyt. Kahn's employment multiplier, as exposited by Keynes (40, p. 116), resulted in the following. Let:

Y' = national income at factor cost

N = total employment, a function of Y'

I = domestic investment

N_2 = domestic investment employment, a function of I

e_e = elasticity of employment in industry as a whole = $\frac{dN}{dY'}\frac{Y'}{N}$

e'_e = elasticity of employment in investment industries = $\frac{dN_2}{dI}\frac{I}{N_2}$

k = investment multiplier

Then:

$$\Delta Y' \cong \frac{Y'}{e_e N}\Delta N = \frac{dY'}{dN}\frac{N}{Y'}\frac{Y'}{N}\Delta N = \frac{dY'}{dN}\Delta N$$

$$\Delta I \cong \frac{I}{e'_e N_2}\Delta N_2 = \frac{dI}{dN_2}\frac{N_2}{I}\frac{I}{N_2}\Delta N_2 = \frac{dI}{dN_2}\Delta N_2$$

so that

$$\Delta N \cong \frac{dN}{dY'}\Delta Y' \cong \frac{dN}{dY'} k\,\Delta I \cong \frac{dN}{dY'} k \frac{dI}{dN_2}\Delta N_2$$

Therefore, Kahn's employment multiplier is

$$\frac{\Delta N}{\Delta N_2} \cong \frac{dN}{dY'} k \frac{dI}{dN_2}$$

For the Keynesian formulation of the Kahn employment multiplier to be adaptable to the multiplier investment and multiplier product accounts developed in the preceding section of this chapter, the symbols used by Keynes have to be redefined.

Let: Y' = GUP produced - transfer derivative income
N = total - transfer derivative employment
I = multiplier investment (includes exports)
N_2 = multiplier investment employment
k = the corresponding investment multiplier

Kahn's employment multiplier can then be stated in the form given by Keynes for purposes of this thesis.

A problem arises as to how multiplier investment employment is to be estimated. At the urban economy aggregation level, all output is either consumption or multiplier investment. The only alternative is to estimate N_2 to be the same fraction of N that I is of Y'. No matter how I is defined, there are difficulties unless all the output of certain industries goes to investment and others supply none to investment. If the latter condition prevails, then N_2 employment is the employment of these particular industries. For this reason, it will be hazarded, Keynes spoke of investment industries.

In checking over the St. Louis transactions matrix, it was found that no industry had all of its output going to investment, instead varying proportions of their output went to investment activities. A large part of output may not have gone to consumption or investment but have gone to other firms as intermediate input. The reason for distinguishing intermediate input is exactly the same as that for distinguishing intermediate imports discussed in the "Income Accounts" section of this chapter. This case is considered to be general by the writer. In such a situation, the output of industries has to be broken down as to its disposition by local buyers in order to estimate N_2 employment. To do this, the value of output going to investment as a proportion of total value produced has to be used to estimate N_2 as the same proportion of total employment by any given industry. It was in order to be able to estimate N_2 by this method that the regional investment multiplicand account was set up as the value of output going to investment activities and the usual negative components of investment; taxes, inventory depletions, consumption imports and intermediate imports were cast as components of the savings (leakage) function.

D. Hoyt Multiplier

Homer Hoyt's employment multiplier is concerned with the export portion of multiplier investment as defined for the regional version of Kahn's multiplier. N_2 is now divided into two mutually exclusive and exhaustive classes, N_e associated with exports and N_d associated with the remainder of multiplier investment. Exports will be denoted by E and the remainder of multiplier investment by D. N_2 now equals $N_e + N_d$. To get from $\Delta N/\Delta N_2$ to $\Delta N/\Delta N_e$, the following derivation is presented. As before

$$\Delta N \cong \frac{dN}{dY'} \Delta Y' \text{ and } \Delta Y' \cong k \, \Delta I.$$

Now

$$\Delta I = \Delta(D + E) \cong \frac{\partial(D + E)}{\partial N_d} \Delta N_d + \frac{\partial(D + E)}{\partial N_e} \Delta N_e.$$

Therefore,

$$\Delta N \cong \frac{dN}{dY'} k\left[\frac{\partial I}{\partial N_d} \Delta N_d + \frac{\partial I}{\partial N_e} \Delta N_e\right]$$

so that

$$\frac{\Delta N}{\Delta N_e} \cong \frac{dN}{dY'} k\left[\frac{\partial I}{\partial N_d} \frac{\Delta N_d}{\Delta N_e} + \frac{\partial I}{\partial N_e}\right].$$

This is Homer Hoyt's employment multiplier in formal terms. The right hand side of the approximate equality will be referred to as the Hoyt employment multiplier <u>model</u>.

It will be noted that the Hoyt model supposes knowledge of the ratio $\Delta N_d/\Delta N_e$. If this is not the case, then some assumption must be made as to the ratio. When the estimators of the Hoyt multiplier are examined in Chapter IV, this will be seen to be precisely the case.

E. Induced Investment

In the Kahn and Hoyt employment multipliers just developed, multiplier investment was autonomous. These two employment multipliers will now be developed for the case in which there is induced domestic investment. This dependent component of total multiplier investment will respond to changes in GUP produced minus transfer derivative income. GUP produced minus transfer derivative income will be denoted by Y' as before.

To develop the investment multiplier, planned total investment, denoted by I', is made equal to I + g(Y'). "Planned" refers to the GPCF + inventory addition + induced investment portion of I'. Planned saving is assumed to be a linear function of Y' and is made equal to L + (1/k)Y' where k is defined as previously and "L" is any constant. Deriving the investment multiplier by equating planned investment and planned saving, it follows that:

$$I + g(Y') = L + (1/k)Y'$$

$$I - L = \frac{1}{k}Y' - g(Y')$$

$$\Delta(I - L) = \Delta\frac{1}{k}Y' - \Delta g(Y')$$

$$\cong \left(\frac{1}{k} - \frac{dg(Y')}{dY'}\right)\Delta Y'$$

so

$$\Delta Y' \cong \frac{k\,\Delta(I - L)}{(1 - k\,g'(Y'))} \quad \text{where} \quad g'(Y') = \frac{dg(Y')}{dY'}$$

usually thought of as

$$\Delta Y' \cong \frac{k\,\Delta I}{(1 - k\,g'(Y'))}.$$

For Kahn's multiplier, ΔI is again written approximately equal to

$$\frac{dI}{dN_2}\,\Delta N_2 \text{ and } \Delta N \cong \frac{dN}{dY'}\,\Delta Y'$$

then

$$\Delta N \cong \frac{dN}{dY'}\left(\frac{k}{1 - k\,g'(Y')}\right)\frac{dI}{dN_2}\,\Delta N_2$$

and

$$\frac{\Delta N}{\Delta N_2} \cong \frac{dN}{dY'}\left(\frac{k}{1 - k\,g'(Y')}\right)\frac{dI}{dN_2}.$$

If $g(Y')$ is a positive function and if $k\,g'(Y') < 1$ when $Y' > 0$, then this employment multiplier is larger than or equal to that in the noninduced case for any given original level of N or N_2.

Because of the nature of the employment multiplier approximation, it will be the same value for ΔN_2 of all sizes from any given original employment level. However, assume $g(Y')$ to be a monotonic increasing positive function of Y'. If $g(Y')$ is a polynomial of degree greater than one, the employment multiplier will increase as the level from which ΔN_2 is measured increases, ceteris paribus. If $g(Y')$ is linear and homogeneous, the employment multiplier is invariant to the original N_2 level, ceteris paribus. If $g(Y')$ is less than linear in Y', the employment multiplier will decrease as the original N_2 level increases, ceteris paribus.

For Hoyt's employment multiplier, the only change is as in Kahn's, from k to $k/(1 - k\, g'(Y'))$.

F. Input-Output

The input-output scheme used in the St. Louis study by Professor Werner Hirsch will be described so as to expedite the empirical comparison of the Hoyt and Leontief employment multiplier estimates for St. Louis.

The idea in Leontief input-output analysis is to determine, after empirically finding the quantities applying to an input-output transactions matrix, linear homogeneous input-output functions for certain sectors of an economy referred to as processing sectors. All other sectors of the economy are termed final demand sectors. Assuming the input-output coefficients of the processing sectors to be appropriate, the total output of these processing sectors can then be determined for any desired amount and sector distribution of final demand. The method by which the processing sector input-output coefficients and output are determined, is indicated in the following paragraphs.

In filling in the input-output transactions matrix of

	Business processing													
	·	Construction	...	Trade	...	Service	·	Households	Local govt	State and federal govt	Inventory additions	GPCF	Exports	Output
·														
Construction														
⋮														
Trade														
⋮														
Service														
·														
Households														
Local govt														
State and federal govt														
Inventory depletions														
GPCF														
Imports														
Input														

Figure 1. Input-output transactions matrix

Figure 1, business processing sectors such as manufacturing components, trade and service have their sales to the indicated sectors, for a period of time such as a year, distributed across the row at producers prices including taxes. The output of manufacturing concerns which is not physically processed any further is entered as sales to households, if it is eventually sold to local consumers by trade firms, in the input-output table. Only the gross margin on such sales experienced by wholesalers and retailers is included in their output figure sold to consumers. Purchases and business taxes of the processing sectors are allocated down the columns. Construction activity classified as private investment is assigned to the gross private capital formation (GPCF) column. Households include all factor payments before direct taxes, except corporate profits which are included after taxes, plus depreciation allowances for their row entries. Household column entries are consumption items, personal taxes and savings. The latter item is allocated to the GPCF row. Household investment in residences, which appears in the construction row, is placed in the GPCF column. Similarly, sales of business processing sectors utilized for private investment purposes are entered in the GPCF column. Government output consists of taxes and government input consists of purchases of industry output and labor services plus transfer payments. Inventory change consists of depletions for the row entry and additions for the column. The GPCF column consists of all real investment sales by the sectors involved. The final row and column are imports and exports respectively. They consist of out of the area imports and exports. The state and federal government and export sectors, as defined in the St. Louis input-output study, are combined into one sector termed "exports" in the following chapters.

Gross urban product would consist of: (1) household consumption purchases including direct imports and indirect taxes as entered in the household column, omitting the direct personal taxes and savings entries, (2) all local government expenditures for industrial output and labor which would be the local government column minus transfers and direct imports, (3) urban (domestic) investment which would be the gross private capital formation column less direct imports plus inventory additions minus inventory depletions and (4) net foreign investment equal to exports plus state government and federal government local purchases excluding state and federal transfers and direct imports minus consumption and intermediate imports. The actual 1955 St. Louis transactions matrix yields only gross urban product produced plus government transfer payments because government transfer payments were not indicated separately. This amount was $4,694,508,000. Gross urban product would be that received if gross area factor owner flows are included in imports and exports and gross urban product produced if it is omitted.

In an open input-output model, some of the columns of the input-output matrix previously presented would be classified as final demand items. Items so considered might be government, gross private capital formation, households and exports. It is the open formulation which leads to multipliers from which sector outputs can be estimated by multiplying final demand.

The mathematical formulation of an open input-output model of "n" processing sectors is presented in the following. Working towards the multiplier matrix the input-output matrix entries are put in the following form:

$$\begin{array}{ccccccccc}
(X_1 - x_{11}) & - & x_{12} & - & \cdots & - & x_{1n} & = & y_1 \\
-x_{21} & + & (X_2 - x_{22}) & - & \cdots & - & x_{2n} & = & y_2 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
-x_{n1} & - & x_{n2} & - & \cdots & + & (X_n - x_{nn}) & = & y_n
\end{array}$$

where X_i is an algebraic variable denoting the total output in dollars of processing industry i ($i = 1, 2, \ldots, n$).

x_{ij} is the output of industry i used as an input by industry j, actual amounts determined from empirically implementing an input-output transactions matrix.

y_i is an algebraic variable denoting the amount used by the nonprocessing segments of the economy which constitute final demand.

Total production of any particular processing industry (i) must, in equilibrium equal its production required for final demand plus that amount of its output required by the other processing industries. Using the preceding notation, this may be written

$$X_i - \sum_{j=1}^{n} x_{ij} = y_i$$

or

$$X_i - \sum_{j=1}^{n} a_{ij}X_j = y_i$$

where $a_{ij} = x_{ij}/X_j$ is a technical coefficient which states the amount of industry i output needed as an input by industry j per dollar of output by industry j. X_j is the observed total of any processing industry input-output column, less any inventory depletions when computing a_{ij}. Once a_{ij} is computed, X_j becomes an algebraic variable identical to its corresponding X_i. This will give n linear equations in n unknowns. By writing out the latter formulation of the n equations, it can be seen that they set up in matrix notation as

$$[I - A]X = y$$

so that

$$X = [I - A]^{-1}y$$

where

A denotes the matrix $[a_{ij}]$ of input-output coefficients

X' denotes the vector X_1, X_2, ..., X_n of processing sector outputs

y' denotes the vector y_1, y_2, ..., y_n of final demand

Each element of $[I - A]^{-1}$ shows how much of industry (i) output is directly and indirectly required per dollar of output for final demand by industry (j). Evans and Hoffenberg (26) worked with the transpose of $[I - A]^{-1}$ in their input-output study of the U.S. for 1947. Then each row element of $[I - A]^{-1}$ shows how much of industry j output is needed per dollar of output by row industry i for final demand. For purposes of context, if

$$X = [I - A]^{-1}y, \text{ then } X' = y'[I - A]^{-1}.$$

The inverse of $[I - A]$ sums all the interdependent outputs in an input-output scheme in the same sense in which $1/(1-r)$ sums $1 + r + r^2 + \ldots$ for the investment multiplier.

G. Leontief Multiplier

Final demand sectors are now defined to consist of households, all government, gross private capital formation and exports. "y" will be thought of as the change in dollar final demand. Multiplying $[I - A]^{-1}$ times y gives the value of production required of the processing industries. Converting these production figures into employment by use of sector output-employment functions gives the total change in employment. The final demand production-employment change for the jth industry, for example, is referred to as the direct change. Other processing sector production changes arising from the changes in final demand for the jth industry are given by the product of the other jth column elements of $[I - A]^{-1}$ times the change in final demand for the jth industry. These are referred to as indirect production changes and, when converted into employment changes, indirect employment changes. This indirect or linkage relationship gives rise to a linkage employment multiplier when direct and indirect employment change is divided by direct employment change. Such sector employment multipliers were presented in Hirsch's study of St. Louis.

If households are moved from the final demand of the previous formulation to processing, then consumption no longer appears in final demand which now consists of all government, GPCF and exports. For a sector that was in processing with both formulations of final demand, say X_i, its final demand component will be smaller if there was any household consumption of its output in the previous formulation of final demand. The X_i is unchanged but is now equal to the ith row of the inverse of an augmented $[I - A]$ times a y which has been augmented by one element but whose other elements have been reduced by the amount of their output used for consumption. The augmentation to the dimension of the final demand vector consists of household output derived from exports (= 0 for gross urban product produced), gross private capital formation (= 0) and government (> 0). It has been shown by F. V. Waugh (75) that $[I - A]^{-1} = I + A + A^2 + A^3 + \ldots$ where A is the technical coefficient input-output matrix previously discussed. If A is augmented by a row and column whose elements are not all zero, then it can be seen that each element of an augmented $[I - A]^{-1}$ must be larger than or equal to its unaugmented $[I - A]^{-1}$ element.

Since processing sector outputs are invariant to the sector composition of final demand, the reduction in final demand employment caused by shifting household consumption from final demand to processing must in general yield linkage plus income-consumption effect sector employment multipliers which are larger than the linkage multipliers. Subtracting direct and indirect employment changes associated with a given change in final demand for a particular industry will isolate the income-consumption production and employment effects, referred to as induced effects. Linkage plus induced effect employment multipliers can be calculated by dividing total employment change by direct employment change. This was not done in the St. Louis input-output study.

Inclusion of households in the processing matrix means that linear homogeneous sector consumption functions are being used. This is thought to overstate the induced effect. An alternative to this would be to substitute marginal coefficients of consumption in the technical coefficient matrix before inverting. Marginal coefficients for the nation might have to be used since urban consumption functions, particularly by industry division, are not usually available.

III. COMPARISONS OF EMPLOYMENT MULTIPLIER MODELS

The purpose of this chapter is to investigate some conditions sufficient for the Kahn, Hoyt and Leontief employment multipliers to give equal estimates of total employment change (ΔN) to be expected from a change in their respective multiplicand employments. Conditions for the equality of their employment multiplier magnitudes will also be examined.

A. Kahn and Hoyt

Kahn's employment multiplier has been stated as

$$\frac{\Delta N}{\Delta N_2} \cong \frac{dN}{dY'} \, k \, \frac{dI}{dN_2}$$

and Hoyt's as

$$\frac{\Delta N}{\Delta N_e} \cong \frac{dN}{dY'} \, k \left[\frac{\partial I}{\partial N_d} \frac{\Delta N_d}{\Delta N_e} + \frac{\partial I}{\partial N_e} \right].$$

For the two estimates of total employment change (ΔN) to be always equal, the following relation must hold:

$$\frac{dN}{dY'} \, k \, \frac{dI}{dN_2} \Delta N_2 = \frac{dN}{dY'} \, k \left[\frac{\partial I}{\partial N_d} \frac{\Delta N_d}{\Delta N_e} + \frac{\partial I}{\partial N_e} \right] \Delta N_e$$

which reduces to

$$\frac{dI}{dN_2} \Delta N_2 = \frac{\partial I}{\partial N_d} \Delta N_d + \frac{\partial I}{\partial N_e} \Delta N_e.$$

The simplest way to realize the latter required identity is for all the functional expressions involved on the two sides of the identity to be constants. Then I must be a linear function (f) of N_2. Let GPCF + inventory additions + local govt G, denoted by D, and exports including supragovt G, denoted by E, be a function of their accompanying employment N_d and N_e. Then $I = D + E$ must also be a linear function of N_d and N_e where $N_d + N_e = N_2$.

Let

$$I = \alpha + \beta_2 N_2$$

then

$$\frac{dI}{dN_2} = \beta_2.$$

To evaluate $\partial I/\partial N_d$ and $\partial I/\partial N_e$ a composite function must be evaluated. Diagrammatically:

$$\begin{array}{ccc} & D & — & N_d \\ I \begin{matrix} \nearrow \\ \searrow \end{matrix} & & & \\ & E & — & N_e \end{array}$$

So that

$$\frac{\partial I}{\partial N_d} = \frac{\partial I}{\partial D}\frac{dD}{dN_d} = 1 \cdot \frac{dD}{dN_d} = \frac{dD}{dN_d}$$

and

$$\frac{\partial I}{\partial N_e} = \frac{\partial I}{\partial E}\frac{dE}{dN_e} = 1 \cdot \frac{dE}{dN_e} = \frac{dE}{dN_e}.$$

Therefore, for $\partial I/\partial N_d$ and $\partial I/\partial N_e$ to be constants, D and E must be linear functions of N_d and N_e respectively.

Let

$$\begin{aligned} D &= m\alpha + \beta_d N_d \\ E &= (1 - m)\alpha + \beta_e N_e \end{aligned} \qquad \text{where } 0 \leq m \leq 1$$

In order that

$$\frac{dI}{dN_2} \Delta N_2 = \frac{\partial I}{\partial N_d} \Delta N_d + \frac{\partial I}{\partial N_e} \Delta N_e,$$

$$\beta_2 \Delta N_2 = \beta_d \Delta N_d + \beta_e \Delta N_e.$$

To solve for ΔN_d and ΔN_e, given ΔN_2, β_d, β_e and β_2, the following two equations in ΔN_d and ΔN_e must be satisfied:

$$\beta_d \Delta N_d + \beta_e \Delta N_e = \beta_2 \Delta N_2$$

$$\Delta N_d + \Delta N_e = \Delta N_2$$

The first equation states I to be a linear function of $N_d + N_e$. If $\beta_d = \beta_e = \beta_2 = \beta$, then the two equations are linearly dependent and there are an infinite number of solutions for ΔN_d and ΔN_e. If $\beta_d \neq \beta_e$, the unique solution for ΔN_d and ΔN_e is:

$$\begin{bmatrix} \beta_d & \beta_e \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} \beta_2 \Delta N_2 \\ \Delta N_2 \end{bmatrix}$$

$$\begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} \beta_d & \beta_e \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_2 \Delta N_2 \\ \Delta N_2 \end{bmatrix}$$

If the output-employment function f is not linear, then the possible equality of the estimated ΔN's by Kahn's and Hoyt's multipliers becomes doubtful.

With linear output-employment functions, the Hoyt multiplier is equal to that of Kahn if $\beta_d = \beta_e = \beta_2$ and if ΔN_d is zero. If $\beta_2 = \beta_d$ or β_e when $\beta_d \neq \beta_e$, then ΔN_e or ΔN_d, respectively, are identically zero and this case will not be considered. Generally, the Hoyt multiplier will be larger than the Kahn, assuming the regression coefficients in I, D and E, expressed as functions of their respective employments, to be positive and larger than zero. This is demonstrated in the following.

Let $\beta_d = a$, $\beta_e = b$, $\beta_2 = c$, where $\beta_d < \beta_2 < \beta_e$ or $\beta_d > \beta_2 > \beta_e$. From

$$\begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} \beta_d & \beta_e \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_2 & \Delta N_2 \\ & \Delta N_2 \end{bmatrix}$$

substituting and letting $\Delta N_2 = 1$

$$= \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} c \\ 1 \end{bmatrix}$$

$$= \frac{1}{a - b} \begin{bmatrix} 1 & -b \\ -1 & a \end{bmatrix} \begin{bmatrix} c \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{a - b} & \frac{-b}{a - b} \\ \frac{-1}{a - b} & \frac{a}{a - b} \end{bmatrix} \begin{bmatrix} c \\ 1 \end{bmatrix}$$

$$\Delta N_d = \frac{c}{a - b} - \frac{b}{a - b} = \frac{c - b}{a - b};$$

if $a < b$, then $a < c < b$ and $\frac{c - b}{a - b} > 0$. If $a > b$, then $a > c > b$ and $\frac{c - b}{a - b} > 0$.

$$\Delta N_e = \frac{-c + a}{a - b}$$

and by the same argument it is > 0 whether $a \lessgtr b$. Letting ΔN_2 be free to vary gives

$$\Delta N_d = \frac{\Delta N_2 (c - b)}{a - b}$$

$$\Delta N_e = \frac{\Delta N_2(-c + a)}{a - b}$$

So, ΔN_d and ΔN_e are always of the same algebraic sign and the sign is that of ΔN_2. It remains to show that the Hoyt $\Delta\hat{N}/\Delta N_e$ is greater than the Kahn $\Delta\hat{N}/\Delta N_2$ when $\beta_d \neq \beta_e$. From the preceding expressions for ΔN_d and ΔN_e it can be seen that neither of them will equal zero unless they both do. The conditions imposed guarantee that the Kahn $\Delta\hat{N}$ will equal the Hoyt $\Delta\hat{N}$. Therefore, with ΔN_2 not equal to zero

$$\frac{\Delta N}{\Delta N_2} = \frac{\Delta N}{\Delta N_d + \Delta N_e} < \frac{\Delta N}{\Delta N_e}.$$

B. Kahn and Leontief

In discussing the employment multiplier of Leontief with reference to the other two employment multipliers, the vocabulary of the different models will sometimes be used interchangeably for ease of exposition. For example, "multiplicand" may be used as a synonym for "final demand" and vice versa, "linked" for "indirect" and so on. The synonym use of the multiplier and input-output terms should be clear from the context.

The argument to be used in demonstrating some conditions sufficient for equality of the ΔN's estimated by the Leontief and the other two models will not be mathematical. Instead, knowledge of the models will be used to deduct in a more verbal manner, what the conditions are.

There are four main types of relations to be specified in comparing the Leontief with the Kahn and Hoyt employment multiplier models. They are:

(1) final demand changes

(2) investment multiplier

(3) output-employment functions

(4) linkage: a synonym for intermediate input-output relations

(1) Changes in final demand for the Leontief model will be confined to changes in:

exports, including supra government expenditures in the area minus transfer payments

urban government expenditures minus transfer payments

gross private capital formation

gross inventory additions

These Δ (final demand) figures include all of their indirect imports in the input-output formulation. In the Kahn model, the multiplicand consists of exactly the same items.

(2) The Kahn employment multiplier aggregates changes in final demand into one sum and applies the investment multiplier to it to estimate Δ(investment) + Δ(consumption) - $\Delta(M_c + M_{inter})$ - Δ(inventory depletions). The Leontief model applies the inverse matrix to the Δ(final demand) to estimate sector outputs from which Δconsumption, Δimports and Δ(inventory reductions) can be computed. Since the multiplicands are

the same for both models, the investment multipliers must be specified so that the Kahn product will equal the Leontief Δinvestment + Δconsumption - $\Delta(M_c + M_{inter})$ - Δ(gross inventory reductions). The input-output scheme used for empirical purposes in this thesis uses linear homogeneous sector leakage functions. So, for any particular sector relative distribution of Δ(final demand), the investment multiplier will be the same value from all levels of GUP. The over-all leakage function for a Kahn model must be linear so that the Kahn investment multiplier will be a constant throughout the range of GUP. The corresponding Kahn leakage function would be expected to be linear and homogeneous. Therefore, in order to make an unconfounded comparison of the two employment multipliers, the sector relative distribution of Δ(final demand) for the Leontief multiplier must be fixed to accommodate the less flexible Kahn model. With these conditions, the Δ(GUP produced minus transfer derivative income) given by the Kahn and Leontief models should be equal.

(3) Linear output-employment functions would be the simplest to work with to ensure equality of the ΔN's and ΔN_2's estimated by the Kahn and Leontief employment multipliers from equal ΔY's' and Δ(final demand). Subjecting an input-output model to linear sector output-employment functions is not sufficient to guarantee that a function aggregating the sectors would be linear unless (a) the sector regression coefficients

are equal or (b) the relative frequency distribution of the sector final demand and output is fixed. The restrictions of case (a) need not be bothered with since the sector relative distribution of Δ(final demand) and hence of Δ(output) has already been specified to attain equality of the Kahn and Leontief investment multipliers. The result is that the Leontief Δ(final demand) and Δ(output) will be linear homogeneous functions of their corresponding aggregate employments, ΔN_2 and ΔN, and these functions could be computed by weighting the input-output sector regression coefficients by the specified sector relative distribution of ΔN_2 and ΔN, respectively.

In comparing the Kahn and Leontief employment multipliers, linear output-employment functions have been specified. There is a difference, however, in the makeup of the total output-employment functions which apply to the two models. The same difference also exists between the Hoyt and Leontief total output-employment functions. Total output in the Kahn model consists of value-added in the private sectors of the economy plus government expenditures on the four ultimate inputs in the area. In the Leontief model, processing sector outputs include the value of inputs purchased from other local industries and imports so that the Leontief total output amount is not primarily value-added. This causes the Kahn and Leontief total output-employment linear regression coefficients to be different even though all other relevant factors are specified so as to not cause any difference.

(4) If the firms supplying the output classified as final demand purchase inputs from local firms, then linkage employment of the type defined in Chapter II exists. Although all linkage employment must of necessity be included in ΔN, the interesting thing is that in the Leontief employment multiplier model, linkage employment is not included in the sector ΔN_2's. Kahn's employment multiplier contains no mention of linkage employment. It has been described as though the only inputs were the four ultimate inputs of land, labor, capital and risk taker plus imports. For purposes of homology with the Leontief employment multiplier the Kahn $\Delta Y'$ and ΔI, set up as functions of ΔN and ΔN_2 respectively, must include all linkage employment in ΔN and none in ΔN_2.

It will be noted that the employment of government sectors is omitted from the ΔN and ΔN_2 of the processing sectors of the Leontief employment multiplier, as formulated. The reason for this is that all government is included in final demand, hence there are no government rows in $[I - A]^{-1}$ with which to multiply the changes in final demand to estimate government outputs. Therefore, there are no government Δ(final demand) and government Δ(output) figures to convert into employment. This difficulty can be surmounted but a more serious problem is that government Δ(final demand) and government Δ(output) are taxes and government employment is directly related, not to taxes, but to non-transfer payments to households. Government payments to households are included in final demand and constitute a part of household output. Thus far, this is the only part of household output that represents an amount of employment not already included in the employment of the business processing sectors. Employment in the household output would be equal to the government employment in final demand and the household sector employment multiplier would have to be regarded as equal to one. Some government establishments, such as utilities, which sell output directly to buyers could be included in the processing sectors of an input-output scheme. In such a case, the employment multiplier pertaining to this government employment could be greater than one. Because of data limitations, all government activity will be considered as confined to the government sectors in this thesis. Civilian government employment has already been included in the Kahn and Hoyt employment multiplier models and will also be included in the Leontief model. All local government expenditures are considered a part of local final demand and all supra government expenditures are considered to be a part of exports.

Members of the Armed Forces stationed in an area could be treated as residents which would include them in government employment and include their pay in the household row of an input-output table. Or, they could be regarded as nonresidents and only their purchases in the area as export sales would enter into a Leontief employment multiplier scheme. Military employment will be excluded from the Kahn, Hoyt and Leontief employment multipliers in this thesis. The employment of domestics has not been mentioned heretofore. There is, strictly speaking, no employment multiplier relationship of the type being discussed in this thesis pertaining to household domestics and this employment will be omitted from the Kahn, Hoyt and Leontief employment multipliers also.

The input-output scheme of Leontief is usually used to present sector employment multipliers. Kahn presents only one over-all employment multiplier. To compare the two models, an over-all Leontief multiplier can be computed by dividing the summation of the sector total employment changes by the summation of the final demand employment changes. This is the same as weighting each Leontief sector employment multiplier (j) by

$\Delta N_{2j} / \sum_{i=1}^{n+2} \Delta N_{2i}$ where i = 1, 2, ..., n+2 identifies the sectors
n+1 is local government
n+2 is supra government

Subject to the preceding conditions, both the Kahn and Leontief multiplicands should represent the same amount of employment change and their products represent the same total employment change. Therefore, the estimated ΔN's and the two employment multipliers should be equal.

C. Hoyt and Leontief

To compare the Hoyt and Leontief employment multipliers, the formulation of final demand for the input-output model will be defined to consist of the same components as in the Kahn-Leontief comparison of the preceding section. Then, in computing an over-all Leontief employment multiplier for comparison with a Hoyt multiplier, only the employment change relating to exports will be included in the denominator of an over-all adjusted Leontief employment multiplier. Other final demand employment change will be included in ΔN, of course.

The problem of the sector distribution of Δ(final demand) in the Kahn-Leontief comparison was handled by specifying that the sector relative distribution be fixed. One reason for this requirement was to ensure correspondence of the investment multipliers in the two models. For the same reason, the Δ(final demand) sector distribution must be specified in making a Hoyt-Leontief comparison.

Because the Hoyt employment multiplier is stated in terms of the ratio of total employment change to export employment change, the ratio $\Delta D/\Delta E$ and the sector relative distribution of the two components of the Δ(final demand) must be specified to have equality of the Hoyt and adjusted over-all Leontief employment multipliers. This must be done, subject to meeting the total final demand relative distribution requirements needed for correspondence of the two investment multipliers.

With the ratio and sector relative distribution of ΔD and ΔE specified, the sector relative distribution of Δoutput is also fixed. Sector linear output-employment functions can then be aggregated into Hoyt Y', D and E linear functions of N, N_d and N_e respectively. N_d and N_e are defined to exclude linkage employment. With these conditions, the total and export employment changes given by the Hoyt and adjusted over-all Leontief employment multiplier models, hence their employment multipliers, should be equal. If ΔD is specified equal to zero, then no adjustment would have to be made in computing a Leontief over-all employment multiplier that would be equal to Hoyt's. If an unadjusted over-all Leontief employment multiplier were calculated, it would be smaller than the Hoyt if ΔD, hence ΔN_d, were not equal to zero, although the estimated ΔN's would still be equal. This is the same conclusion reached in the Kahn-Hoyt comparison.

D. Kahn, Hoyt and Leontief

To achieve equality of the ΔN's estimated by the three employment multiplier models considered, a number of sufficient conditions have been placed on the determining relationships involved in the multiplier models. These included:

(1) The definition of total savings to consist of net personal savings, net corporate savings, depreciation, taxes, out-area transfers, gross inventory reductions, direct imports

of consumption goods and imports of intermediate inputs.

(2) Total savings was expressed as a function of GUP produced plus transfers.

(3) Specification of the sector relative distribution of investment, which, from the nature of the Leontief input-output scheme, caused total savings to be a linear homogeneous function of GUP which resulted in the investment multiplier being a constant.

(4) Specification of the ratio $\Delta D/\Delta E$ and the ΔD, ΔE sector relative distributions subject to meeting the $\Delta D + \Delta E = \Delta I$ sector relative distribution in (3).

(5) Linear output-employment functions for the sectors were used and, with the preceding restrictions, for the Kahn and Hoyt sector aggregates.

With these conditions, given a change in final demand, all three employment multipliers will predict the same change in total employment. The Kahn and unadjusted over-all Leontief multiplier will be equal and the Hoyt will be greater than or equal to them.

IV. ESTIMATORS OF HOYT'S EMPLOYMENT MULTIPLIER

Two estimators of Hoyt's multiplier that are frequently used in the literature on the subject will be considered here. The formulas with which export and investment multiplier total employment are used to produce an estimate of the Hoyt employment multiplier are what is meant by the term "estimators of Hoyt's multiplier". Knowledge of export employment and multiplier total employment will be assumed in this chapter. The next chapter will consider methods of estimating these two employments.

A frequently used estimator of the Hoyt multiplier, $\Delta N/\Delta N_e$, is the ratio N/N_e determined for one period of time, usually a year. The other form utilizes the regression coefficient from the fitted linear regression of N on N_e, usually for only two time periods, as an estimator of $\Delta N/\Delta N_e$. Each form can use a definition of N_e that includes or excludes indirect export employment. To illustrate what is meant by indirect export employment, if local company A sells a portion of its output to local company B which exports a portion of its output, then some of the employment in company A is regarded as indirect export employment. Indirect export employment is often referred to as linked export employment. Both estimators, with linked and unlinked definitions of N_e, will be collated with the Hoyt employment multiplier model as discussed in Chapters II and III for purposes of assessing the characteristics they imply about the relationships implicit and explicit in the Hoyt employment multiplier model.

A. $N/N_e = \Delta N \hat{/} \Delta N_e$

The nonregression estimator N/N_e of $\Delta N/\Delta N_e$ will be referred to as the static estimator. This estimator will be examined for some sufficient restrictions it imposes upon the functions involved in the model of the Hoyt multiplier in order to agree with the model, $\Delta N/\Delta N_e \cong \frac{dN}{dY'}k\left[\frac{\partial I}{\partial N_d}\frac{\Delta N_d}{\Delta N_e} + \frac{\partial I}{\partial N_e}\right]$. This then means that the right hand side of the previous approximate equality is equal to N/N_e.

The simplest way for the Hoyt multiplier model to equal N/N_e is that the functions involved in the model be of the following general type:

$$\text{Savings} = \frac{1}{k}\, Y'$$

$$\text{Investment} = I = D + E$$

$D = \beta\ N_d$ and $E = \beta\ N_e$ and by necessity $I = \beta\ N_2$

$$Y = \beta'\ N$$

The equilibrium Y' is given by

$$I = \frac{1}{k}\, Y'$$

$$Y' = kI$$

substituting $$\beta' N = k(D + E)$$

$$N = \frac{1}{\beta'}k(\beta N_d + \beta N_e)$$

$$= \frac{dN}{dY'} k(\frac{\partial I}{\partial N_d} N_d + \frac{\partial I}{\partial N_e} N_e)$$

So that, when in equilibrium

$$\frac{N}{N_e} = \frac{dN}{dY'} k(\frac{\partial I}{\partial N_d} \frac{N_d}{N_e} + \frac{\partial I}{\partial N_e})$$

The right hand side of the preceding equality is the expression for the Hoyt employment multiplier model stated in Chapter II, except that N_d/N_e appears in place of $\Delta N_d/\Delta N_e$. It will be observed that there is no approximation involved for the Hoyt model in this case. Since information on non-export investment, D, is not ordinarily gathered in making Hoyt estimates, this estimator supplies a needed statement about the ratio $\Delta N_d/\Delta N_e$, namely that it be the same as the average ratio N_d/N_e. Since $\Delta N_d/\Delta N_e = N_d/N_e$ no matter what the level of I, this indicates that N_d/N_e is a constant. (In the examination of the Hoyt employment multiplier model in Chapter II, there were an infinite number of solutions for ΔN_d and ΔN_e, given ΔN_2, in the case where $\beta_d = \beta_e = \beta_2$.) It is also indicated that the sector relative distributions of ΔN_d and ΔN_e be the same as the sector relative distributions of N_d and N_e respectively, since the investment multiplier is a constant and $\Delta N_d/\Delta N_e$ is substituted for N_d/N_e. Since the output-employment functions are linear, specifying sector relative distributions of ΔN_d and ΔN_e and the ratio $\Delta N_d/\Delta N_e$, will specify one and only one corresponding sector relative distribution for ΔD, only one for ΔE, and only one $\Delta D/\Delta E$ ratio. The fact that a linear output-employment function is single valued will not be repeated.

If the output-employment functions have different regression coefficients, then the following is sufficient for the model $\Delta N/\Delta N_e$ to equal N/N_e.

Let
$$D = \beta_d N_d$$
$$E = \beta_e N_e$$
$$Y' = \beta' N$$

Then, with the same savings function and investment total as before, the equilibrium Y' is given by:

$$I = \frac{1}{k} Y'$$
$$Y' = kI$$

substituting

$$\beta' N = k(D + E)$$
$$N = \frac{N}{Y'} k(\beta_d N_d + \beta_e N_e)$$
$$\frac{N}{N_e} = \frac{N}{Y'} k(\beta_d \frac{N_d}{N_e} + \beta_e)$$
$$= \frac{dN}{dY'} k(\frac{\partial I}{\partial N_d} \frac{N_d}{N_e} + \frac{\partial I}{\partial N_e})$$

The right hand side of the preceding equality is the expression for the Hoyt employment multiplier model stated in Chapter II, except that N_d/N_e appears in place of $\Delta N_d/\Delta N_e$. Thus, this estimator supplies the same assumption about the ratio $\Delta N_d/\Delta N_e$ when the D and E regression coefficients differ, namely that it be the same as the average ratio N_d/N_e. Again the sector relative distributions of ΔN_d and ΔN_e must be specified to be that of N_d and N_e since the investment multiplier is a constant and $\Delta N_d/\Delta N_e$ substitutes for N_d/N_e. The Hoyt model is again exactly equal to $\Delta N/\Delta N_e$.

A more powerful argument as to $\Delta N_d/\Delta N_e$ being a constant is the following:

Let

$$I = \beta_2 N_2$$

$$D = \beta_d N_d$$

$$E = \beta_e N_e$$

Then the following must hold

$$\begin{bmatrix} \beta_d & \beta_e \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} \beta_2 & \Delta N_2 \\ & \Delta N_2 \end{bmatrix}$$

for $\Delta N_2 = 1$, we have

$$\begin{bmatrix} \beta_d & \beta_e \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} \beta_2 \\ 1 \end{bmatrix}$$

substituting

$$\begin{bmatrix} D/N_d & E/N_e \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} I/N_2 \\ 1 \end{bmatrix}$$

Triangularizing

$$\begin{bmatrix} 1 & \frac{N_d}{D} \frac{E}{N_e} \\ 0 & 1 - \frac{N_d E}{N_e D} \end{bmatrix} \begin{bmatrix} \Delta N_d \\ \Delta N_e \end{bmatrix} = \begin{bmatrix} \frac{N_d}{D} \frac{I}{N_2} \\ 1 - \frac{N_d I}{N_2 D} \end{bmatrix}$$

$$\Delta N_e = \frac{DN_2 - IN_d}{DN_2} \cdot \frac{DN_e}{DN_e - EN_d}$$

$$= \frac{D(N_d + N_e) - (D + E)N_d}{DN_2} \cdot \frac{DN_e}{DN_e - EN_d}$$

$$= \frac{(DN_e - EN_d)DN_e}{(DN_e - EN_d)DN_2}$$

$$= \frac{N_e}{N_2}$$

$$\Delta N_d = (\frac{I}{N_2} - \frac{EN_e}{N_eN_2})\frac{N_d}{D} = (\frac{D + E - E}{N_2})\frac{N_d}{D} = \frac{D}{N_2} \cdot \frac{N_d}{D}$$

$$= \frac{N_d}{N_2}$$

Therefore, for all ΔN_2,

$$\frac{\Delta N_d}{\Delta N_e} = \frac{\frac{N_d}{N_2}\Delta N_2}{\frac{N_e}{N_2}\Delta N_2} = \frac{N_d}{N_e}$$

Although I has not been specified to be a linear homogeneous function of N_d and N_e, this must be so as shown in the following:

$$I = D + E = \beta_d N_d + \beta_e N_e \text{ with } N_d/N_e = \sigma$$

substituting $I = (\beta_d \sigma + \beta_e)N_e$ a linear homogeneous function.

If N_e is defined to include linked export employment, then the only change in the preceding discussion of the estimator N/N_e is that it is lowered if any linkage exists. This is accomplished in the estimator and model by decreasing the β_d and β_e.

The N/N_e equal to the model $\Delta N \hat{/} \Delta N_e$ argument depended upon, among other things, the fact that for the Hoyt model $\Delta \hat{N}/\Delta N_e$ to equal N/N_e, N/N_e had to be a constant. When induced investment is present, the only case in which this remains true is when the induced investment function $g(Y')$ is linear and homogeneous in Y'. When this is the situation, the investment multiplier is still a constant over the total investment range and N/N_e can still be identical with $\Delta N/\Delta N_e$.

B. Regression Estimator

A second estimator form frequently used in estimating the Hoyt employment multiplier is the linear regression of N on N_e. The regression coefficient is used as an estimator of the Hoyt multiplier. For the regression coefficient β'' in $N = \delta + \beta'' N_e$ to be identical to the Hoyt model expression, the functions involved in the model most simply would be of the following type:

$$\text{Savings} = L + \frac{1}{k} Y'$$

$$\text{Investment} = I = D + E$$

with

$$D = m\alpha + \beta_d N_d$$

$$E = (1 - m)\alpha + \beta_e N_e$$

$$Y' = \gamma + \beta' N$$

$$0 \leq m \leq 1, \quad \beta_d \lesseqgtr \beta_e, \quad \alpha \geq 0, \ \gamma \geq 0.$$

The latter two conditions follow because D and E, as defined, cannot be less than zero and N, N_d and N_e cannot be less than zero. The equilibrium Y' is given by

$$I = L + \frac{1}{k} Y'$$

$$Y' = k(I - L)$$

Substituting

$$\gamma + \beta' N = k(D + E - L)$$

$$= k\left[m\alpha + \beta_d N_d + (1 - m)\alpha + \beta_e N_e - L\right]$$

$$= k(\alpha - L + \beta_d N_d + \beta_e N_e)$$

$$N = \frac{-\gamma}{\beta'} + \frac{k}{\beta'}(\alpha - L + \beta_d N_d + \beta_e N_e)$$

and if

$$N_d/N_e = \sigma$$

$$N = \frac{-\gamma}{\beta'} + \frac{k}{\beta'}(\alpha - L + \beta_d \sigma + \beta_e) N_e$$

let

$$\delta = \frac{-\gamma + k(\alpha - L)}{\beta'}$$

$$\beta'' = \frac{k}{\beta'}(\beta_d \sigma + \beta_e)$$

then

$$N = \delta + \beta'' N_e$$

and

$$\Delta N = \beta'' \Delta N_e$$

so that $$\frac{\Delta N}{\Delta N_e} = \beta''$$

It remains to establish the identity of β'' with the Hoyt employment multiplier model.

$$\beta'' = \frac{k}{\beta'}(\beta_d \sigma + \beta_e)$$

$$= \frac{dN}{dY'} k\left(\frac{\partial I}{\partial N_d} \frac{\Delta N_d}{\Delta N_e} + \frac{\partial I}{\partial N_e}\right)$$

which is the expression for the Hoyt employment multiplier model.

Since the investment multiplier in the regression estimator is a constant, this indicates that the sector relative distributions of ΔN_d and ΔN_e and the $\Delta N_d/\Delta N_e$ ratio are constants. Because of the $N_d = \sigma N_e$ substitution involved in deriving the regression estimator, the ΔN_d and ΔN_e ratio and their sector relative distributions must, in a Leontief input-output sense, be identical to that of N_d and N_e. The sector relative distribution of ΔN will, by Leontief input-output analysis, be a constant and be the same as that of N. The only way that the sector relative distribution of ΔN could be different from that of N would be if nonlinear input-output relations prevailed. In such a case, however, the aggregate investment multiplier could not be expected to remain a constant. The only advantage gained by the linear nonhomogeneous output-employment functions is that Y', D and E need not be in constant ratios to their respective employments, hence D/E need not be a constant.

The output-employment functions in the regression estimator are not as restricted as in the static estimator since they must be linear but not necessarily homogeneous. The linear functions and the conditions on ΔN_d and ΔN_e ensure that β'' will be the same as the Hoyt employment multiplier model $\widehat{\Delta N/\Delta N_e}$ which will be exactly equal to $\Delta N/\Delta N_2$.

If N_e is defined to include linked export employment, then the only change in the preceding discussion of the linear regression coefficient estimator is that it is decreased if any linkage exists. This is accomplished in the model and the estimator by decreasing the β_d and β_e.

The preceding argument depended upon the fact that for the Hoyt model $\Delta \hat{N}/\Delta N_e$ to be a constant, most simply, all the expressions involved in the model had to be constants. When induced investment is present, the only case in which the investment multiplier is a constant is when the induced investment function $g(Y')$ is linear and homogeneous in Y'. When $g(Y')$ is linear and homogeneous, then $\Delta N/\Delta N_e$ is identical to the regression coefficient in $N = \delta + \beta'''N_e$ where $\beta''' > \beta''$.

C. Employment Multiplier Constancy

A common feature of the two estimators examined is that they present one value for the export multiplier of any particular city from a given employment level, with no quantitative procedure for adjusting the value for export employment changes of different sizes. The constancy of the Hoyt employment multiplier for changes of various sizes in export employment from a <u>given</u> employment level was also a characteristic of the Hoyt model examined in Chapter II, both in the induced and noninduced investment cases. The constancy in the model stemmed from the linear nature of an approximation by the use of differentials. Therefore, the constancy of the estimators of the Hoyt multiplier for export employment changes of various sizes is accepted. It should be noted that if the functional expressions involved in the Hoyt employment multiplier model are constants, then the approximation is exact.

The two estimators of the Hoyt model examined here specify, in addition to the constancy of the employment multiplier for ΔN_e of all sizes from a given level of N_e, that the value of the multiplier is a constant throughout the range of N_e greater than zero for any particular urban economy. This is not a requirement of the Hoyt model. If the functional expressions involved in the model are not constants, or if $\Delta N_d/\Delta N_e$ is not a constant, then the Hoyt employment multiplier would be expected to vary for different levels of N_d and N_e and/or $\Delta N_d/\Delta N_e$ ratios. An empirical estimator has not been developed for these cases which would be in agreement with the Hoyt model.

D. Import Replacing Employment

An interesting feature arises in the treatment of new local output that replaces some goods and services formerly imported. Such import replacements may allow the Hoyt employment multiplier to be applicable. The following procedure applies to the two Hoyt estimators when using an unlinked definition of N_e. All ΔD and ΔE sector relative frequency distribution requirements are assumed to be satisfied.

First, it must be known if the output of local firms that is replacing some goods and services formerly imported into an urban area is utilized directly for exports, GPCF, local government, local consumption or if it is utilized as intermediate input. If the new import replacing output goes directly to exports including supragovernments, then it all results in an increase in export final demand and a Hoyt employment multiplier would be applicable. An increase in D, possibly through a decrease in imports directly utilized, is assumed. If the output is directly utilized by GPCF and/or local government, then none of it constitutes an increase in export final demand and the Hoyt employment multiplier is not applicable. If all the output goes directly to local consumption or intermediate input, this causes a change in the input-output coefficient matrix and a previously determined Hoyt employment multiplier is not really applicable. The new Hoyt employment multiplier would be larger than the previous one, ceteris paribus.

The two estimators using the linked definition of N_e handle import replacing activity the same way as the unlinked, except that there is no case of output replacing imports going to intermediate input. All goods and services produced are assigned to GPCF, local government, exports and consumption.

V. SURVEY ESTIMATION OF EXPORT EMPLOYMENT

Survey estimation of export employment will be defined to be any estimate based at least in part on information gathered from some of the transactors in an urban economy relating directly to the question of their exports. This is in contrast to estimators of export employment based entirely upon auxiliary data not directly related to exports and which is not gotten by direct solicitation from area transactors. Such estimators are referred to as "nonsurvey" in this thesis and are discussed briefly at the end of this chapter. As far as is known to the writer, survey estimates, as performed, have always used the static form of the Hoyt estimator and an unlinked definition of export employment. Some general types of survey methods in use will be briefly indicated.

A. Survey Methods

The representative firm method consists of selecting firms judged to be representative of their various industry members and soliciting information as to what proportion of their sales went to nonresident persons or businesses. The export percentage is applied to the respective industry total employment to estimate export employment for the industry. Investment multiplier total employment is taken to be total employment in the area. This method was first used in 1949 by multiplier total employment is taken to be total employment in the area. Reasons advanced for not soliciting firms in other industries are that they are too numerous or that they do not know their proportion of sales to nonresident organizations and individuals. An example of this type of contention would be in the study, "Working Denver, an Economic Analysis", by the Denver Planning Office (22, p. 155) in which E. T. Halaas of the University of Denver acted as economic consultant in a survey estimate of the Hoyt employment multiplier.

To the writer's knowledge, a probability sample of area transactors has never been used to implement a survey estimate of the Hoyt employment multiplier. An advantage of a probability sample, compared to the census method, is that it would decrease the time and money necessary to prepare an estimate. An advantage when compared to non-census survey methods is that it would permit the use of the mathematics of probability to calculate, from the observations themselves, a measure of the fallibility of the estimate. There are real difficulties in implementing a probability sample of the type needed to make a Hoyt estimate.

A difficulty not mentioned in the preceding survey techniques is the treatment of area government establishments which do not sell output. An attempt is usually made to classify those area government employees as export who provide goods and services for nonlocal residents or organizations.

This would correspond very roughly to area government activities not financed by area taxes and would be confined to supraregional governments. The procedure adopted in the development of the urban income accounts in this thesis was to classify all supragovernment expenditures, hence employment, as export.

An error common to the survey methods of estimating the Hoyt employment multiplier is that investment multiplier total employment is taken to be total employment in the area. Transfer payments, as defined in Chapter II, would have to be equal to zero for this to be correct. As explained in Chapter II, transfer derivative employment must be omitted from investment multiplier total employment because there is not any investment employment corresponding to transfers. It has been suggested by Alexander (1, p. 257) that some area transfer recipients could be counted as export employment. The estimation of transfer recipients would require a special effort in a Hoyt survey.

B. Unlinked N_e Estimator

The estimator of export employment for any firm in a Hoyt survey determination is the per cent export sales are of total sales, times the firm's total employment. This can be set up in mathematical form as:

$$\left(\frac{E}{T}\right)N = \hat{N}_e \qquad \text{where} \qquad \begin{array}{l} E = \text{export sales} \\ T = \text{total sales} \\ N = \text{total employment} \\ N_e = \text{export employment} \end{array}$$

The explicit statement in the preceding estimator of N_e is that export sales are in the same ratio to total sales as export employment is to total employment. This estimator of N_e will be examined for the conditions it requires of a firm in order that it will be without error.

If a firm produces only one product, then the estimator $\frac{E}{T} \cdot N = \hat{N}_e$ poses no difficulties if finished inventory change is equal to zero. This assumption will be made in the remainder of this section.

If a firm produces two or more products, taking the case of two products for an example, then

$$\left(\frac{E}{T}\right)N = \frac{E_a + E_b}{T_a + T_b} \cdot (N_a + N_b) = \hat{N}_e$$

where E_a = exports sales of good A

E_b = exports sales of good B

T_a = total sales of A

T_b = total sales of B

N_a = total employment engaged in production of A

N_b = total employment engaged in production of B

The following must be true for the export employment estimator to be correct:

$$\frac{E_a + E_b}{T_a + T_b} \cdot (N_a + N_b) = \frac{E_a}{T_a} N_a + \frac{E_b}{T_b} N_b$$

This must be so because the export/total sales fraction of a firm is usually for the firm as a whole, not by product, to the writer's knowledge. To determine the conditions sufficient for the latter equality, the following relation will be used. If

$$\frac{a}{b} = \frac{a'}{b'}, \text{ then } \frac{a + a'}{b + b'} = \frac{a}{b} = \frac{a'}{b'}.$$

Proof: let $a = \lambda a'$, $b = \lambda b'$ so that

$$\frac{a}{b} = \frac{\lambda a'}{\lambda b'} = \frac{a'}{b'},$$

therefore

$$\frac{a + a'}{b + b'} = \frac{a(1 + \lambda)}{b(1 + \lambda)} = \frac{a}{b} = \frac{a'}{b'}$$

So, if

$$\frac{E_a}{T_a} = \frac{E_b}{T_b},$$

then

$$\frac{E_a + E_b}{T_a + T_b} \cdot (N_a + N_b) = \frac{E_a}{T_a} N_a + \frac{E_b}{T_b} N_b.$$

This means that if the export/total sales fractions are equal for all the products produced by a firm, the estimator will be correct. Or,

$$\frac{E_a + E_b}{T_a + T_b} \cdot (N_a + N_b)$$

can be written

$$\frac{N_a + N_b}{T_a + T_b} \cdot (E_a + E_b)$$

So, if

$$\frac{N_a}{T_a} = \frac{N_b}{T_b},$$

then

$$\frac{N_a + N_b}{T_a + T_b} \cdot (E_a + E_b) = \frac{N_a}{T_a} E_a + \frac{N_b}{T_b} E_b$$

$$= \frac{E_a}{T_a} N_a + \frac{E_b}{T_b} N_b.$$

This means that if the employment/sales ratios are the same for all products produced by a firm, the estimator will be correct. Either of these conditions is sufficient for the export employment estimator for a firm to be without error.

It is simplest to imagine the employment/total sales ratios being equal for each product that a firm produces in the following manner. The production value (output) functions, with relation to labor input, must be the same for all products and, unless the output of A/output of B ratio is specified equal to one, they must also be linear and homogeneous with respect to labor.

If the export sales/total sales ratios of all products are the same, there need be no restraints on the output-employment functions involved as far as the estimator of N_e

for a firm is concerned. The ensuing discussion will apply equally well to a linked or unlinked definition of N_e.

As indicated in Chapter IV, the static estimator of the Hoyt employment multiplier was formulated with E, D and Y' as linear homogeneous functions of N_e, N_d and N respectively, in order that the estimator agree with the Hoyt model. Collation of the Hoyt and Leontief models indicated that the sector relative distribution and ratios of D, E and Y' and hence of N_d, N_e and N must be constants. Thus, Hoyt sector output-employment functions would be expected to be linear and homogeneous also. The simplest way for the sector output-employment functions to be linear and homogeneous is for the firms comprising each sector to have linear and homogeneous output-employment functions. Thus a firm must have a linear homogeneous output-employment function even if the output ratios of all its products are equal to one or if the export sales ratios are equal.

If the Hoyt output-employment regression coefficients are not equal for all firms within a sector, then the among firm relative distribution of output in exports, other investment and total must be specified so that the sector E, D and Y'-employment functions will be linear and homogeneous. Unless the input-output relations are identical for all firms within a sector, thinking in terms of Leontief input-output analysis, the among firm relative distribution would have to be specified even if the output-employment regression coefficients were equal. Following this line of thought, the within firm among products relative distribution must be specified as well. So, the export/total production value ratio, other investment/ total production value, and total production value ratios of products for a firm have to be specified in order to conform to the Hoyt multiplier static estimator. The export employment estimator used in conjunction with the Hoyt multiplier estimator requires, in addition, that the employment/total production value or the export/total production value ratios be equal for all the products of a firm.

The regression estimator of the Hoyt employment multiplier was more flexible than the static estimator principally in the fact that E, D and Y' were formulated as linear functions, not necessarily homogeneous, of N_e, N_d and N respectively. The sector relative distributions of N_e, N_d and N were required to be constants as were their over-all ratios. Breaking a sector linear output-employment function down by firm, the firm functions would also be expected to be linear. Linkage considerations, in the Leontief input-output sense, would require the within sector among firm relative distribution of N_e, N_d and N and the within firm ratios, by product, to also be specified. The estimator of export employment, applied to such a firm would be, as before $(E/T)N = \hat{N}_e$.

If a firm produces only one product, then $(E/T)N = \hat{N}_e$

again poses no difficulty, assuming no finished inventory changes.

If a firm produces two or more products, taking the case of two products for an example, then

$$(\frac{E}{T})N = \frac{E_a + E_b}{T_a + T_b} \cdot (N_a + N_b) = \frac{E_a}{T_a} N_a + \frac{E_b}{T_b} N_b.$$

If the latter equality is true, then as before

$$\frac{E_a}{T_a} = \frac{E_b}{T_b} \quad \text{and/or} \quad \frac{T_a}{N_a} = \frac{T_b}{N_b}.$$

If the export fraction of total sales is a constant for all products, then no further conditions on the output-employment functions are needed. If only the output/employment ratios are equal, however, additional conditions are needed. Let

$$T_a = a + \beta_a N_a \quad \text{and} \quad T_b = b + \beta_b N_b$$

then

$$\frac{T_a}{N_a} = \frac{a}{N_a} + \beta_a = \frac{T_b}{N_b} = \frac{b}{N_b} + \beta_b$$

Since output and employment are not dependent variables between nonjoint products, the production value/employment ratios must be examined for an identity. This requires

$$N_a = \lambda N_b$$

$$a = \lambda b$$

$$\beta_a = \beta_b$$

so that

$$\frac{a}{N_a} + \beta_a = \frac{\lambda b}{\lambda N_b} + \beta_b = \frac{b}{N_b} + \beta_b$$

For the estimator of a firm's export employment to be correct, either the export/total sales ratios or total sales/employment ratios must be equal for each product produced by a firm in addition to the conditions required by the Hoyt multiplier regression estimator. If only the total sales/employment ratios are equal, then the regression constants and employments pertaining to any pair of products must be in the same ratio and all regression coefficients must be equal.

C. Linked N_e Estimation

To implement a survey estimate of a linked definition of N_e for an urban area would require the following. To learn the local linkage, exporting firms would be asked from which local firms they bought inputs and in what amount for the time period concerned. Then the linked firms would have to be consulted to find out what proportion of their sales went to the exporter so as to estimate their export employment. However, this might not be the entire export linked employment. The first linked company might have bought inputs from another local company. This second linked firm would have to be contacted to find out what proportion of its sales went to the first linked firm. Then it might be that the second linked company bought some of its input from the local exporter. So, the tracing of export linked employment would go round and round. If, as has been suggested by Leven (44, p. 370) the employment of the first few linked firm is included as export linked and the rest neglected, this could amount to approximating the sum of an infinite series by the first several terms. Leontief's input-output scheme is expressly designed for this type of situation, and an exact solution to the export linked employment for any given sector relative distribution of D and E and their amounts can be had. The method will be indicated and applied in Chapter VI.

Part of the interest expressed by some writers in having export employment include linked export employment is attributed to a desire on their part to remove the possible effect on estimates of export employment of differences in the vertical integration of exporting firms. If the sales and employment of firms were broken down by establishment (separate physical facilities) in survey estimates of export employment, the presence or absence of vertical integration would generally not cause any difference in the estimate of total export employment. An exception would be if vertical integration occurred in the same establishment. This procedure would appear to be much cheaper than a linked export employment estimate if it is desired to remove the vertical integration effect. Such a procedure would also eliminate some of the qualification about the estimator of export employment, as used in Hoyt surveys, indicated in the preceding section.

In Leontief input-output employment multiplier determinations, the sales and employment of firms are broken down by sector if separate facilities (establishments) are utilized at the different industry levels. This should ordinarily eliminate the vertical integration effect on Leontief estimates of total export employment.

D. Nonsurvey Estimators

Nonsurvey estimators were defined at the beginning of this chapter. Their most significant feature is that they do not utilize information directly related to export activity in estimating export employment. There are many variations, but the most commonly used nonsurvey estimator of export employment according to Mattila and Thompson (50, p. 217) will be examined here.

Export employment for an industry is estimated by

$$N_{e_i} = N_i - N \cdot \frac{\mu_i}{\mu}$$

where

N_{e_i} = export employment in urban industry i (i = 1, 2, ..., m) is restricted to be greater than, or equal to, zero

N_i = total employment in urban industry i

N = total employment in the urban area

μ_i = total employment in benchmark economy industry i ($i = 1, 2, \ldots, m$), with the benchmark economy including the urban economy and a larger population; for example, the United States

μ = total employment in the benchmark economy

All of the required data is ordinarily available for major urban areas without any survey effort by a person interested in implementing the preceding estimator. This estimator attributes the proportion industry i employment is of total employment in the benchmark economy to be nonexport employment in urban industry i. Any excess employment in urban industry i is taken to be export employment. Summation over i gives total export employment for the urban area.

In the form given, the estimator of export employment for industry i is that of a regression estimator, $\hat{N}_{e_i} = a - bx$. However, the important unknowns are not the regression constant and coefficient, but the x. x is in the form of a ratio estimator:

$$x = \frac{\mu_i}{\mu} = \frac{N_i - \hat{N}_{e_i}}{N} = \frac{\hat{N}_{m_i}}{N}$$

where N_{m_i} = nonexport (local) employment in urban industry i. This ratio estimator is not the type envisioned by statisticians since the ratio estimator is stated to be consistent by Cochran (20, p. 114). If μ_i and μ are known, not estimated, there is no obvious reason why the estimate should be correct. A similar type of ratio estimator was examined earlier in this chapter, $E/T = (N_e/N)$ for a firm. Although this ratio estimator cannot be demonstrated to be consistent statistically, it can be defended on other grounds. For example, it does not seem reasonable to designate employment for a firm as export on any other basis than according to the ratio of exports to total sales, subject to the qualifications of that estimator brought out in the "Unlinked N_e Estimator" section of this chapter.

An estimator of N_{e_i} employing an auxiliary variable that a statistician might use would be the following.

$$\hat{N}_{e_i} = M_i \bar{n}_{e_i} + bM_i(\bar{N}_i - \bar{n}_i)$$

where: the lower case letters denote sample data

the higher case letters denote population data

the bars ($^{-}$) relate to firm averages

M_i = the number of firms in industry i

The procedure would be to sample the firms in industry i, ascertaining their export and total employment. Then, if the relation between export and total employment were expected to be approximately linear, but not homogeneous, and total employment in the industry were known, the above would be appropriate. This estimator is obviously consistent.

A convincing nonstatistical defense of the nonsurvey estimator of export employment for an urban industry examined here has not occurred to the writer. Leven (45, p. 255) investigated the economic conditions necessary for the nonsurvey estimator to be valid and concluded that export employment

VI. SURVEY ESTIMATES

Some of the data obtained from an input-out study of the St. Louis Metropolitan Area for the year 1955 will be used to prepare estimates of the Leontief and Hoyt employment multipliers. Because the data were compiled for only one time period, the Hoyt estimates will be of the static variety as defined in Chapter IV.

Not all firms in the St. Louis Metropolitan Area were included in the St. Louis study. Those firms that were included were selected by a judgement process and for this reason statistical measures of reliability of sector totals could not be presented. Also, trade association and census data were used in preparing the sector totals.

As indicated in Chapter II, transfer derivative income and employment should be excluded in preparing estimates of employment multipliers. Because government transfers could not be separated in the input-output data, the figures were treated as though this amount were of zero magnitude. Since the magnitude was in all likelihood greater than zero, all employment multiplier estimates presented in this chapter are judged to be slightly inflated.

A. Data

Following, in tabular form (Table 1), are some of the data from the St. Louis input-output study used in the preparation of the employment multiplier estimates.

The first column presents the sector codes used by Professor Hirsch in presenting his report. It will be noted that certain of his sectors were combined for purposes of calculation in this thesis. The second column presents 1945 Standard Industrial Classification (SIC) codes which most nearly correspond to the sector outputs. The third column gives a verbal SIC name to the sectors. Column four is domestic investment before inventory reductions are subtracted plus local government expenditures and column five is the export portion of net foreign investment. Column six is the market value of production, termed output in this thesis. Column seven is civilian employment in the metropolitan area. The employment figures exclude domestics, unpaid family workers and the non-agricultural self-employed. The last exclusion was caused by the nature of the available employment data.

B. Leontief Estimate

An adjusted, over-all Leontief employment multiplier estimate was prepared from the data in Table 1. In preparing this estimate, advantage was taken of the fact that if empirically determined input-output coefficients and final demand amount and sector distribution were used, the sector

Table 1. St. Louis metropolitan area data, 1955

Input-output sector[a]	Approximate 1945 SIC code[b]	Sector title[b]	Output value (in 000) Other Inv.[c]	Exports[c]	Total[c]	Employment[d]
1	20	Food	3,101	738,380	1,126,209	33,100
2	22, 23	Textiles, apparel	2,577	107,692	181,522	19,000
4	26	Paper	1,680	41,035	130,668	8,600
5	27	Printing, publishing	3,302	45,842	140,823	13,500
6	28	Chemicals	12,931	420,721	494,752	20,700
7	29	Petroleum and coal	4,251	483,961	657,631	8,100
8	31	Leather	2,044	134,633	148,687	14,900
9	331, 332	Iron, steel	8,522	173,434	275,226	15,600
10	other 33	Nonferrous metals	7	122,477	151,163	8,000
11	34	Fabricated metals	6,176	82,842	223,490	19,900

[a]Adapted from (31).

[b]Adapted from (32).

[c]Source (30b).

[d]Source (52b).

Table 1 (Continued).

Input-output sector[a]	Approximate 1945 SIC code[b]	Sector title[b]	Output value (in 000) Other inv.[c]	Exports[c]	Total[c]	Employment[d]
12	35	Nonelec. machinery	22,104	114,868	233,802	20,100
13	36	Electric machinery	13,246	167,215	202,970	19,900
14	371	Motor vehicles	139,486	247,964	585,402	14,000
15	other 37	Other transp. eqpt.	8	283,507	293,121	19,000
16, 3	01, 21, 24, 25, 30, 32, 38, 39	Miscl. mfg., agric.	44,102	207,216	520,155	37,100[de]
17, 21	48, 49	Utilities, comm.	10,624	48,596	288,343	18,300
18	40	Railroads	10,739	223,507	370,904	24,600
19	41 - 47	Other trsp. service	7,238	62,280	222,192	25,400
20, 26	50 - 59	Trade	53,104	76,442	965,146	155,100
22	60 - 67	Finance, etc.	19,921	16,188	668,002	35,200
23, 24	70 - 78, 80 - 83, 85 - 87, 89	Service	4,377	46,502	535,940	75,200
25	10 - 14, most 19, 79, 84	Mining, ordnance amusement	134,684	236,959	606,638	14,500

[e]Adapted from (73).

Table 1 (Continued).

Input-output sector[a]	Approximate 1945 SIC code[b]	Sector title[b]	(in 000) Output value			Employment[d]
			Other inv.[c]	Exports[c]	Total[c]	
27	15, 16, 17	Construction	458,530	108,310	723,890	42,800
28	88	Households	182,133	393,544	3,868,595	--
29	local 90	Local govt.	--	--	--	20,000
31, 32	supra 90	Supra govt.	--	--	--	42,000
33	--	GPCF	--	--	--	--
Imports	--	Imports	--	--	--	--
Total			1,144,887	4,584,115	--	724,600

output solution could also be that empirically observed at the same time. This is demonstrated in the following.

Original entries in an input-output transactions matrix set up as

$$\begin{matrix} x_{11}, & x_{12}, & \ldots, & x_{1m} \\ x_{21}, & x_{22}, & \ldots, & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1}, & x_{m2}, & \ldots, & x_{mm} \end{matrix}$$

where the mth column can be defined to consist of inventory additions, local government expenditures, GPCF and exports including all supra governmental expenditures and transfer payments. In the St. Louis study, the mth column would be the sum of columns four and five in Table 1. The m - 1 columns to the left of the mth column will consist of the business processing sectors and households. Denoting $\sum_{j=1}^{m} x_{ij}$ by X_i, we can write

$$\begin{matrix} (X_1 - x_{11}) & - & x_{12} & - \ldots - & x_{1m} & = 0 \\ - x_{21} & + & (X_2 - x_{22}) & - \ldots - & x_{2m} & = 0 \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ - x_{m1} & - & x_{m2} & - \ldots + & (X_m - x_{mm}) & = 0 \end{matrix}$$

which is equivalent to

$$\begin{matrix} (1 - a_{11})X_1 & - & a_{12}X_2 & - \ldots - & a_{1m}X_m & = 0 \\ - a_{21}X_1 & + & (1 - a_{22})X_2 & - \ldots - & a_{2m}X_m & = 0 \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ - a_{m1}X_1 & - & a_{m2}X_2 & - \ldots + & (1 - a_{mm})X_m & = 0 \end{matrix}$$

where

$$a_{ij} = \frac{x_{ij}}{X_j} = \frac{x_{ij}}{X_i}$$

since in this formulation, any observed column total is equal to its corresponding observed row total. A certain set of X_i is known to be a solution of this set of equations in any empirical determination of an input-output scheme. In the case of the St. Louis study, it would be the data of column six in Table 1. This means that the $[I - A]$ matrix is singular.

If the mth column of the preceding set of linear homogeneous equations is moved to the right hand side of the equality and the mth row is deleted, we have

$$\begin{bmatrix} (1 - a_{11}) & - a_{12} & - \cdots & - a_{1n} \\ - a_{21} & + (1 - a_{22}) & - \cdots & - a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ - a_{n1} & - a_{n2} & - \cdots & + (1 - a_{nn}) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} x_{1m} \\ x_{2m} \\ \vdots \\ x_{nm} \end{bmatrix}$$

where $n = m - 1$. This is a nonhomogeneous set of linear equations and a solution vector $[X_i]$, $(i = 1, 2, \ldots, n)$, can be the same as the empirical X_i's satisfying the previous linear homogeneous equations. If the reduced $[I - A]$ matrix is nonsingular, the solution vector has to be the same as the empirical X_i's.

In computing an adjusted over-all Leontief employment multiplier, final demand will consist of the same components as the previously discussed column m. Export employment is calculated from the export portion of final demand. For purposes of homology with the static Hoyt estimator, linear homogeneous output-employment functions have to be used. Export employment for each sector can be calculated by multiplying the fraction that export sales are of total output times sector employment. The over-all adjusted Leontief employment multiplier estimate will be equal to total area employment divided by total export employment. From the St. Louis data, the estimate is 724,600/264,928 $\cong$ 2.74.

C. Hoyt Estimates

Four different estimates of the Hoyt employment multiplier will be presented. The first estimate will be an implementation of the Hoyt model that will be in agreement with the Leontief estimate. From Chapter II, the Hoyt model is

$$\frac{\Delta N}{\Delta N_e} \cong \frac{dN}{dY'} k \left[\frac{\partial I}{\partial N_d} \frac{\Delta N_d}{\Delta N_e} + \frac{\partial I}{\partial N_e} \right]$$

In conformance with the static estimator discussed in Chapter IV, this becomes

$$\frac{\Delta N}{\Delta N_e} = \frac{N}{Y'} \cdot \frac{Y'}{D+E} \left[\frac{D}{N_d} \frac{N_d}{N_e} + \frac{E}{N_e} \right]$$

substituting from Table 1 and page 26

$$= \frac{724,600}{4,694,508,000} \cdot \frac{4,694,508,000}{5,729,002,000} \left[\frac{1,144,887,000}{75,038} \cdot \frac{75,038}{264,928} + \frac{4,584,115,000}{264,928} \right]$$

$\cong 2.74$

The N_d employment is estimated by a procedure entirely homologous to that of estimating N_e described in the preceding section.

The preceding estimate of the Hoyt employment multiplier made use of linear homogeneous <u>sector</u> output-employment functions to estimate N_d and N_e. In a usual survey estimate of the Hoyt multiplier, the procedure is to use linear homogeneous firm output-employment functions to estimate N_e. This procedure was used to estimate N_e for those business processing firms reporting in the St. Louis input-output study. Their employment was approximately twenty per cent of St. Louis total business processing employment. For those sector firms not reporting, the residual total sales, export sales and total employment were used to estimate the remaining export employment. This procedure results in an estimate of N_e equal to 260,207, and a multiplier of 2.78. Formulating this in terms of the model and its static estimator:

$$\frac{\Delta N}{\Delta N_e} = \frac{N}{Y'} \cdot \frac{Y'}{D+E}\left[\frac{D}{N_d}\,\frac{N_d}{N_e} + \frac{E}{N_e}\right]$$

$$= \frac{724{,}600}{4{,}694{,}508{,}000} \cdot \frac{4{,}694{,}508{,}000}{5{,}729{,}002{,}000}\left[\frac{1{,}144{,}887{,}000}{75{,}038} \cdot \frac{75{,}038}{260{,}207} + \frac{4{,}584{,}115{,}000}{260{,}207}\right]$$

$\cong 2.78$

There are two conditions, either of which would be sufficient for the estimates of the Hoyt employment multiplier using linear homogeneous output-employment functions for sectors, versus linear homogeneous output-employment functions for firms, to be in agreement. Either every firm in a sector must have the same fraction of its total output going to exports, or the output/employment ratios must be equal for all firms in a sector. These are the same two conditions discussed in the "Unlinked N_e Estimator" section of Chapter V, but at a greater level of aggregation.

Reverting back to the use of linear homogeneous sector output-employment functions to estimate N_e, an estimate of a full export linked N_e can be made. What is needed is a way to exhaust linkage employment by attribution to D, E and C since these terms make up the area activity components of final demand, in a general economic sense. If the household column is moved to final demand in Leontief analysis, the inverse matrix of business processing coefficients gives values which, multiplied times a final demand of D plus E plus C, will result in the empirically observed sector outputs. Any one sector output is equal to its row of inverse coefficients times the final demand vector. Converting output into employment, all processing employment can therefore be attributed to D, E and C. Government employment can be attributed by definition. Sector export and export linked output can be computed by the product of the sector row of the inverse matrix times the export components of the final demand vector. Sector export and export linked employment can be estimated by converting export and export linked output into employment. This method applied to the St. Louis data results in an export employment of 346,342. The full export linked N_e gives a Hoyt employment multiplier estimate of 2.09.

It has been suggested that export linked employment for a firm can be estimated by multiplying the firm's employment by the fraction of its output going to a local firm times the export fraction of total sales of the latter firm and summing over all such latter firms. This might be carried out to several layers of linkage. The difficulty with this method is that it will generally not exhaust all of a firm's linkage employment. However, the method can serve as an approximation.

To assess the adequacy of the approximate method for the St. Louis Hoyt employment multiplier linked estimate, the method was applied on a sector basis for the first layer of linkage as shown by the St. Louis transactions matrix. Data was not available to the writer to prepare the estimate using survey firm data. However, the sector application permits a more meaningful comparison with the previous exhaustive method of linked export employment imputation. The approximate method results in an export and export linked employment figure of 325,075 compared to the previous estimate of 346,342 and an unlinked N_e of 264,928. The approximate export linked

N_e figure results in a Hoyt employment multiplier estimate of 2.23.

To determine an exact linked N_e figure for an urban economy, an input-output table would have to be prepared. Even tracing the first few export linked firms for an approximate linked N_e figure is the beginning of an input-output table. Although the Hoyt employment multiplier is restricted as to the ratio and relative sector distribution of ΔD, ΔE and Δoutput while the Leontief multiplier is not, the main advantage of the Hoyt multiplier would seem to be the greater ease of a survey estimate. If an approximate linked export employment definition is used, this advantage disappears as the approximation grows more exact. For this reason, and for homology with the Leontief employment multiplier, the unlinked definition of N_e would seem preferable for the Hoyt multiplier.

VII. CONCLUSION

This thesis has investigated an economic parameter known as the export employment multiplier with reference to urban economies. The urban export employment multiplier has been referred to as the Hoyt employment multiplier in this thesis after the name of its principal innovator, Homer Hoyt. Much literature on the subject refers to this concept as the basic employment multiplier, where the word "basic" is used as a rough synonym for export.

A model for the export employment multiplier, similar to the Keynesian model of the investment employment multiplier was developed. These multiplier models assume that investment can be written as a function of investment employment, that total area income is a function of investment and that total employment is a function of total area income. Therefore, total employment is indirectly a function of investment employment. The export employment multiplier relates total employment to export employment and requires the knowledge, or an assumption, as to the ratio of other investment employment to export investment employment. A third type of employment multiplier is the input-output model developed by Leontief. By collation with the investment and input-output employment multipliers, conditions as to the industrial composition of changes in all investment and export investment, input-output relations including the consumption and leakage functions, and output-employment functions were developed so that the same change in total employment would be indicated by the three employment multiplier models in a given situation.

The Hoyt employment multiplier model presented treated investment, including exports, as the determinant of area income. The only provision for area income influencing the level of investment was an induced investment function which related to investment other than export. An induced investment function for exports would indicate that the level of income in an area influenced its level of exports. This phenomenon is sometimes referred to as "feedback". As mentioned by Tiebout (69, p. F7), the St. Louis area probably does not experience feedback to any appreciable extent.

Two estimators of the Hoyt model are in use. One assumes that the ratio of change in total employment to the change in export employment is the same as the average ratio and was referred to as the static estimator. The other estimator assumes that total employment is a linear function of export employment, the ratio of total employment change to export employment change is therefore equal to the linear regression coefficient. All survey determinations of the Hoyt multiplier have utilized the static estimator to the writer's knowledge. The difference between the two estimators is that the functional relations involved in the Hoyt model are assumed to be linear and homogeneous by the static estimator and only linear by the regression estimator. For either estimator, the change in export, all investment employment and all employment, by industry, must be the same as the corresponding total industry employment relative frequency distributions.

To implement either the static or regression estimators of the Hoyt multiplier, estimates of export and investment multiplier total employment are required. The survey estimator of export employment, by firm or industry, assumes export employment to be in the same ratio to total employment as export sales are to total sales. For this to be true, there must be no change in finished inventories. Also, if more than one product is produced, either the export/total sales ratios or the employment/sales ratios, by product, must be equal.

Nonsurvey estimators of export employment abound in profusion. Generally, they estimate export employment by differences in the relative frequency distribution of employment by industry in the area studied compared to the distribution for some larger area. Such estimators cannot be evaluated statistically as they rely entirely upon auxiliary data to estimate export employment. Because there is no compelling nonstatistical defense of such estimators, and other weaknesses, they were not considered more than briefly in this thesis.

Investment multiplier total employment is usually taken to be total employment in the area. In this thesis, the Hoyt multiplier was cast in terms of income produced and persons employed in the area, rather than income received and employed residents. For this reason, area employment arising from government transfers and private in-area transfers should be omitted from total employment as there is no investment employment corresponding to these transfers, hence an employment multiplier does not exist.

Data from an input-output study of St. Louis for 1955 by Professor Werner Hirsch was utilized to make estimates of the Hoyt multiplier and the Leontief adjusted over-all employment multiplier. The Hoyt estimator was necessarily of the static form. With the Leontief employment multiplier restricted to linear homogeneous output-employment functions and the sector relative frequency distribution of investment the same as that observed in preparing the input-output transactions matrix, the Leontief employment multiplier was the same as the Hoyt. The only unusual condition necessary for this equality was that observed inventory additions were left in investment and inventory reductions were part of the leakage function. These two items are usually omitted in static equilibrium analysis but had to be included here to insure the equality because the Hoyt estimators have no way of adjusting for inventory changes. In the particular case at hand, the change in the export employment multiplier would have been slight. Government transfer derivative employment was included in both the Hoyt and Leontief estimates as the original transfer payments could not be identified.

The definition of export employment to include local export linked employment was investigated. A complete imputation of linkage employment to consumption and investment, including exports and local government, can be made. Likewise, an estimate of export employment including all linked export employment can be made. If such an estimate of export employment for a Hoyt export employment multiplier is desired, then an input-output table must be prepared and utilized. This would entail considerably more work than that involved in a usual Hoyt survey estimate. Since a Leontief employment multiplier is considerably more flexible as to the industry relative frequency distribution of investment than are the Hoyt estimators, it would be expected that one would shift to the Leontief employment multiplier if such a table were prepared. An interesting point is that Leontief employment multiplier estimates apparently always use an unlinked definition of final demand employment, export in this case. If an approximate estimate of export linked employment were made in a Hoyt survey by tracing some of the export linkage, then as the approximation grew more exact, the labor of preparing an input-output table would be in some respects surpassed without gaining any of the final demand flexibility of the Leontief multiplier. For this reason, it is concluded that an unlinked definition of export employment would be advisable for Hoyt multiplier surveys. It is also concluded that estimating unlinked export employment by establishment, rather than firm, would reduce the effect of changes or differences in vertical integration.

A frequent topic of conjecture about the Hoyt employment multiplier is its variability for the same city over time and its difference between cities at the same point in time. Considering the multiplier as measured by a survey implementation of the static estimator for the same city over time, any one of the following might contribute to its variation. The investment multiplier embodied in the export multiplier is an equilibrium concept. Even though a measurement may be attributed to the employment multiplier at any time, an urban economy need not necessarily be in equilibrium; that is, the planned portion of investment need not equal the actual components and planned leakages need not equal actual leakages. Changes might occur in the sector relative frequency distribution of investment or in the ratio of other investment to export investment. Either of these could cause differences in a Hoyt static employment multiplier estimate. Input-output functions, including households, and output-employment functions are assumed to be linear and homogeneous. If they are not of this nature, then as an economy moved along the true functions, the static estimator could give varying estimates. Changes in these functions, even if linear and homogeneous could cause variation. Discrepancies in the definition of export employment over time might occur. If total export workers are estimated on the basis of sample data, sampling variation could account for changes in the employment multiplier estimate. The usual inclusion of transfer payment derivative employment in computing the multiplier could cause variation if the ratio of this employment to export employment changed. Changes in vertical integration could also affect estimates of the Hoyt employment multiplier if unlinked export employment were estimated by firm rather than establishment.

To investigate possible sources of differences in the Hoyt employment multiplier among cities at the same point in time, as measured by the static estimator, sources attributable to the functions implied to be linear and homogeneous not being so or not being the same linear and homogeneous functions, to export employment definitional differences, to sampling variation in estimating total export employment or to differences in the ratio of transfer payment derivative employment to export employment will be mentioned first. The most obvious cause of differences in export employment multipliers would then be differences in the relative magnitude of exports in the investment total and/or in the sector distribution of exports. Differences in vertical integration could influence Hoyt employment multiplier estimates if unlinked export employment were estimated by firm rather than establishment. Not all of the possible causes of inter and intra city differences in Hoyt employment multiplier estimates need be cumulative. Some could be offsetting and thereby cause Hoyt estimates to be closer together than they otherwise would be.

The temptation arises to speak of the export employment in a city as supporting other employment, as though other employment does not earn its keep. This writer would consider it more accurate to say that investment employment, preponderantly export in the case of St. Louis, tends to determine the location of other employment. Another attribute of export employment as indicated by Weimar and Hoyt (76, pp. 31-32), is that quick growth possibilities for a city would appear to lie in expanding such employment.

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